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Division of Engineering
BROWN UNIVERSITY
PROVIDENCE, R. I.

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A LINE SOURCE ON AN INTERFACE BETWEEN TWO MEDIA

BY

V. M. PAPADOPOULOS

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ABSTRACT

The velocity and pressure field of an unsteady line source is calculated by a similarity method when its strength is taken to have a step-function time dependence. The acoustic approximation is used in the case when the source lies on the stationary plane interface between homogeneous fluids of different density; the velocity of sound in each fluid is also different.

The time derivative of the solution in section 1 of this acoustic problem gives the velocity potential when an acoustic line pulse occurs in the surface of, say, a deep sea. This is a first order solution in which the interaction of the acoustic disturbance above and below the interface is examined.

The solution in section 1 is also of direct application in electromagnetic theory. This is discussed in section 3.

AF 4561/5

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INTRODUCTION

In some recent work Craggs (1956, 1957) and Papadopoulos (1959a, 1959b) have shown the value of the assumption of dynamic similarity in the solution of a number of unsteady two-dimensional problems in various physical situations. In each case the unknown quantity satisfies the wave equation.

In this paper we shall determine the nature of the field of a uniform line source which is suddenly set up at some definite moment on the plane interface between two different homogeneous fluids. We assume a linearized equation of state, and that the source is weak enough for the acoustic approximation to be valid. We take the source to be at the origin $r=0$, and we take the time $t=0$ to be the moment at which the source is made active. Under the assumption that the subsequent unsteady motion is irrotational it is well known (see e.g. Friedlander 1958) that the velocity potential satisfies the wave equation $\nabla^2\phi(r,\theta,t) = \frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2}$ where c is the velocity of sound in the medium at rest, while the pressure change p and the particle velocity $\underline{q}$ satisfy the equations

$$p = \partial\phi/\partial t \qquad , \qquad \rho\,\underline{q} = -\underline{\nabla}\,\phi \tag{1}$$

Here ρ refers to the constant density in an undisturbed medium.

Within a single uniform medium, it is known (see e.g. Lamb 1932) that the potential of a line source of uniform density $U(t)$ ($U(t) = 0$ if $t<0$, $U(t) = 1$ if $t>0$) is $\frac{1}{2\pi}\operatorname{sech}^{-1}(r/ct)$; it is clear from this result that it is reasonable to assume in the present problem that the velocity potential depends only on two variables $s(=r/t)$ and θ . This is the assumption of dynamic similarity. It may be added that the pressure corresponding to the above potential is identical with Hadamard's elementary solution (1923) of the wave equation in two dimensions.

AF 4561/5

Suppose now that some quantity $S(s,\theta)$ satisfies the wave equation in the variables (r,θ,t). Then since $s=r/t$, it follows that S must satisfy the equation

$$s^2\left(1-\frac{s^2}{c^2}\right)\frac{\partial^2 S}{\partial s^2} + s\left(1-\frac{2s^2}{c^2}\right)\frac{\partial S}{\partial s} + \frac{\partial^2 S}{\partial \theta^2} = 0 . \tag{2}$$

If $s>c$, the equation is hyperbolic. Put $s= c \sec u$, so that

$$\frac{\partial^2 S}{\partial u^2} - \frac{\partial^2 S}{\partial \theta^2} = 0, \tag{3}$$

and

$$S=f(u-\theta) + g(u+\theta) , \tag{4}$$

where f and g are arbitrary functions, and the lines on which $u+\theta$ and $u-\theta$ are constant are characteristic lines tangent to the circle $s=c$. If $s<c$, the equation 2 is elliptic. Put $s=c \operatorname{sech} (-v)$ so that

$$\frac{\partial^2 S}{\partial v^2} + \frac{\partial^2 S}{\partial \theta^2} = 0 . \tag{5}$$

It follows from equation 5 that with the harmonic function S in the elliptic region we may introduce a conjugate $T(v,\theta)$, so that $W=S + iT$ is an analytic function, and such that

$$\frac{\partial S}{\partial v} = \frac{\partial T}{\partial \theta} , \quad \frac{\partial S}{\partial \theta} = -\frac{\partial T}{\partial v} . \tag{6}$$

In section 1 the detailed solution of the acoustic problem is given. In section 3 is described the change to be made to give the results relevant in the setting up of a charged line or of a line current.

AF 4561/5

Section 1

In Figure 1, we depict the physical situation in the (s,θ) plane. The upper half of this plane, $0<\theta<\pi$, represents the region occupied by a medium 1, and the lower half, $0>\theta>-\pi$, that occupied by a medium 2. The density ρ and the sound velocity c are distinguished by the suffix 1 or 2 where appropriate. We assume that $c_1 > c_2$. The semicircles $s=c_1$, $s=c_2$ separate the elliptic and the hyperbolic regions in each medium.

There are three principal requirements on the solution of our problem. The first is that the solution of the steady problem shall be approached in the limit as $s \to 0$ (i.e. as $t \to \infty$). The second is that at the interface both the pressure and the normal component of velocity shall be continuous, and the third is that the interface shall remain fixed.

Suppose that the quantity S and the velocity potential are related by the equation $\rho c^2 S = \phi$. From the equations 1 it follows that the radial and transverse components (q_r, q_θ) of the velocity, and the pressure p, satisfy the equations

$$\begin{aligned} \frac{\partial S}{\partial s} &= -\frac{t q_r}{c^2} = -\frac{p t}{\rho c^2 s}, \\ \frac{\partial S}{\partial \theta} &= -\frac{r q_\theta}{c^2}. \end{aligned} \tag{7}$$

Put $m=c_2/c_1$, $k=\rho_2/\rho_1$, then the continuity conditions may be written in the form

$$m^2 k \frac{\partial S_2}{\partial s} = \frac{\partial S_1}{\partial s}, \tag{8}$$

and

$$m^2 \frac{\partial S_2}{\partial \theta} = \frac{\partial^2 S_1}{\partial \theta} \tag{9}$$

We may refer again to Figure 1 to discuss some of the properties of

AF 4561/5

the solution. In the hyperbolic region, it is clear that the value of S as $s \to \infty$ corresponds to the initial value of S. Hence S is uniformaly zero at infinity for all values of θ and from the nature of the solution 4 it follows that the value of S is everywhere zero outside the region AFDE. Within the triangle CDG the solution is necessarily of the form $S_2 = f(u-\theta)$, and in the triangle ABH the solution must be of the form $S_2 = g(u+\theta)$, where f and g are functions to to be determined. The solution must be symmetric about the vertical axis in Figure 1, hence we need only examine the field in the right-hand half of the (s,θ) plane.

Consider the region $s \leq c_1$, $0 \leq \theta \leq \pi/2$. In this region

i) the line OE is a line of symmetry on which $\partial S_1 / \partial \theta = 0$.

The arc ED, which is the envelope of the characteristics in the hyperbolic region, is itself a characteristic. Across this arc the pressure and the radial velocity will be discontinuous. The tangential velocity component must be continuous, however, so that

ii) on ED, $\partial S_1/\partial\theta = 0$.

The continuity of pressure and of normal velocity across the interface CD implies that on CD

$$\frac{\partial S_1}{\partial s} = m^2 k \frac{\partial S_2}{\partial s} = m^2 k \left(\frac{\partial u_2}{\partial s}\right)\frac{\partial S_2}{\partial u_2} = -m^2 k \left(\frac{\partial u_2}{\partial s}\right)\frac{\partial S_2}{\partial \theta} = -k\left(\frac{\partial u_2}{\partial s}\right)\frac{\partial S_1}{\partial \theta} = k\left(\frac{\partial u_2}{\partial v_1}\right)\frac{\partial T_1}{\partial s}$$

Thus $\quad \dfrac{\partial S_1}{\partial s} - k \dfrac{\partial T_1}{\partial s}\left(\dfrac{\partial u_2}{\partial v_1}\right) = 0 \quad$ so that

$$Rl\left\{ \frac{\partial W_1}{\partial s}\left(1 + ik\frac{\partial u_2}{\partial v_1}\right)\right\} = 0. \tag{10}$$

On physical grounds we may expect singularities in W_1 only at the points O, C, D and E. The first quadrant in the circle in the (s,θ) plane corresponds to a semi-infinite strip in the complex (v, θ) plane, and under the transformation $\zeta_1 = \xi_1 + i\eta_1 = \operatorname{sech}(v_1 + i\theta)$ we may map this strip conformally into the upper half of the complex ζ_1-plane. The conditions just enumerated are that

i) singularities are to be expected only at the points $\zeta_1 = 0, m, 1$ and ∞,

ii) $\partial W_1/\partial \zeta_1$ is imaginary, for $\xi_1 = 0$, $\eta_1 > 0$,

iii) $\partial W_1/\partial \zeta_1$ is imaginary, for $\eta_1 = 0$, $|\xi_1| > 1$ and

iv) for $\eta_1 = 0$, $m < \xi_1 < 1$

$$\partial W_1/\partial \zeta_1 = R(\zeta_1) \Big/ 1 - \frac{i}{mk}\left(\frac{\zeta_1^2 - m^2}{1 - \zeta_1^2}\right)^{1/2},$$

where $R(\zeta_1)$ is a function which must take real values on this segment of the real axis. Since $s = c_1$, $\theta = \pi/2$ is an ordinary point both for S and for $\partial S/\partial\theta$ it follows that as $|\zeta_1| \to \infty$

v) $\partial W_1/\partial \zeta_1 = 0(\zeta_1^{-2-\delta})$ with $\delta > 0$.

As $\zeta_1 \to 0$, the field must approach the steady state value, and therefore

vi) $\partial W_1/\partial \zeta_1 = 0(1/\zeta_1)$ for $\zeta_1 \to 0$.

Thus after applying these conditions we may write

$$\frac{\partial W_1}{\partial \zeta_1} = \frac{F(\zeta_1)}{\zeta_1\left[(1-\zeta_1^2)^{1/2} - \frac{i}{mk}(\zeta_1^2 - m^2)^{1/2}\right]}; \qquad (11)$$

it is implied that $F(\zeta_1)$ is bounded as $|\zeta_1| \to \infty$, is real on the positive real axis for $\xi_1 > m$, and is real on the imaginary axis. This final condition implies that F must be an even function of ζ_1.

Whatever may be the formula for $F(\zeta_1)$ which we shall determine, we must, in setting up the solution for medium 2, satisfy the continuity conditions across OC. If in this elliptic region we use the conformal mapping $\zeta_2 = \xi_2 + i\eta_2 = \operatorname{sech}(v_2 + i\theta)$ to bring the region of interest, $s < c_2$, $0 > \theta > -\pi/2$, into the fourth

AF 4561/5

quadrant of the ζ_2 -plane, then across OC, $\zeta_1 = m\,\zeta_2$, and the continuity conditions are

$$m^2 k \frac{\partial S_2}{\partial \zeta_2} = \frac{\partial S_1}{\partial \zeta_1}$$

and (12)

$$m^2 k \frac{\partial T_2}{\partial \zeta_2} = k \left(\frac{1 - m^2 \zeta_2^2}{1 - \zeta_2^2} \right)^{1/2} \frac{\partial T_1}{\partial \zeta_2} .$$

It is clear under the conditions imposed that $F(\zeta_1)$ must be real on the whole of the real axis. If $F(\zeta_1)$ is complex on the real axis for $0 < \zeta_1 < m$ there must be branch points of F at the points $\zeta_1 = 0$ and $\zeta_1 = m$. Hence for this region we may write

$$F(\zeta_1) = A(\zeta_1) + i B(\zeta_1) \left[\frac{\zeta_1^2}{m^2 - \zeta_1^2} \right]^{1/2} , \qquad (13)$$

where $A(\zeta_1)$ and $B(\zeta_1)$ are even functions of ζ_1 which are real on the whole of the real axis and which are bounded as $|\zeta_1| \to \infty$. The continuity conditions (12) lead to the equation

$$m^2 k \frac{\partial W_2}{\partial \zeta_2} = \frac{A(m\zeta_2) + i k B(m\zeta_2) \left[\zeta_2 (1 - m^2 \zeta_2^2)^{1/2} / 1 - \zeta_2^2 \right]}{\zeta_2 \left\{ (1 - m^2 \zeta_2^2)^{1/2} + \frac{1}{k} (1 - \zeta_2^2)^{1/2} \right\}} \qquad (14)$$

This result derived for real values of ζ_2 with $0 < \zeta_2 < 1$, may be continued analytically into the whole of the fourth quadrant of the ζ_2 -plane. If the function B exists, expression (14) has a simple pole at $\zeta_2 = 1$ so that W_2 has a discontinuity at this point. At the corresponding point $\zeta_1 = m$, W_1 has no discontinuity (from equations 11 and 13); the only way to avoid this inconsistency is to set $B \equiv 0$. It follows that $F(\zeta_1)$ is a regular function bounded at infinity and real on the real axis, so that it can only be a real constant A.

We may now write the explicit results,

$$\frac{\partial W_1}{\partial \zeta_1} = \frac{A}{\zeta_1\left\{(1-\zeta_1^2)^{1/2} - \frac{i}{mk}(\zeta_1^2-m^2)^{1/2}\right\}},$$

and (15)

$$m^2 k \frac{\partial W_2}{\partial \zeta_2} = \frac{A}{\zeta_2\left\{(1-m^2\zeta_2^2)^{1/2} + \frac{1}{k}(1-\zeta_2^2)^{1/2}\right\}}.$$

The velocity components and the pressure, which are related to the derivatives of S as in the equations 7, may now be found. Thus for $s < c_1$, in medium 1, when $\zeta_1 = \operatorname{sech}(v_1 + i\theta)$ and $s = c_1 \operatorname{sech}(-v_1)$,

$$\frac{\partial S_1}{\partial s} = \frac{-A}{s\left(1-\frac{s^2}{c_1^2}\right)^{1/2}} \, Rl\left\{1 - \frac{i}{mk}\left(\frac{\zeta_1^2-m^2}{1-\zeta_1^2}\right)^{1/2}\right\}^{-1}, \quad (16)$$

$$\frac{\partial S_1}{\partial \theta} = A \, I_m \left\{1 - \frac{i}{mk}\left(\frac{\zeta_1^2-m^2}{1-\zeta_1^2}\right)^{1/2}\right\}^{-1},$$

and for $s < c_2$ in medium 2, when $\zeta_2 = \operatorname{sech}(v_2 + i\theta)$ and $s = c_2 \operatorname{sech}(-v_2)$

$$m^2 k \frac{\partial S_2}{\partial s} = \frac{-A}{s\left(1-\frac{s^2}{c_2^2}\right)^{1/2}} \, Rl\left\{\frac{1}{k} + \left(\frac{1-m^2\zeta_2^2}{1-\zeta_2^2}\right)^{1/2}\right\}^{-1}, \quad (17)$$

and

$$m^2 k \frac{\partial S_2}{\partial \theta} = A \, I_m \left\{\frac{1}{k} + \left(\frac{1-m^2\zeta_2^2}{1-\zeta_2^2}\right)^{1/2}\right\}^{-1}.$$

The constant A is a measure of the volume of fluid produced by the line source; by considering the steady state in the limit $s \to 0$ or $\zeta \to 0$ we find that the volume created in each medium is $\pi k A c_1^2/1+k$ in medium 1 and $\pi A c_1^2/1+k$ in medium 2. Thus the eventual strength of the source is Ac_1^2; the fluid then produced is apportioned between the two media in the inverse ratio of the densities so that the mass of fluid produced is the same in each medium.

To determine the velocity components and the pressure in the hyperbolic region CDG, we use the explicit results derived from equation 16 and 17 for points on the boundary CD, and we use the characteristic form of the solution in CDG to find the complete result. Thus in CDG $S_2 = f(u-\theta)$ where $s/c_2 = \sec u$; therefore $\partial S_2 / \partial \theta = -f'(u-\theta)$. For c_2 s c_1

$$f'(u) = -\frac{1}{m^2}\left(\frac{\partial S_1}{\partial \theta}\right)_{\theta=0} = -\frac{A}{m^3 k}\left(\frac{s^2-c_2^2}{c_1^2-s^2}\right)^{1/2} \Big/ 1 + \frac{1}{m^2 k^2}\left(\frac{s^2-c_2^2}{c_1^2-s^2}\right), \tag{18}$$

$$= H(s), \text{ say}$$

It follows that in CDG ;

$$\partial S_2 / \partial \theta = - H(s^x)$$

where

$$s^x / c_2 = \sec(u_2 - \theta) = s\left[c_2 \cos\theta + (s^2-c_2^2)^{1/2} \sin\theta\right]^{-1}. \tag{19}$$

Similarly the derivative $\partial S_2/\partial s$ is given by the equation

$$\frac{\partial S_2}{\partial s} = \frac{H(s^x)}{s\left(\frac{s^2}{c_2^2} - 1\right)^{1/2}}. \tag{20}$$

Section 2

As far as fluid motion is concerned the analysis of section 1 is merely an exercise in setting up a quantity which has the property of dynamic similarity in a region with the properties given. By assuming uniform densities for two media, we are of course neglecting gravity, but it is not clear whether we can neglect this effect at the interface. Under the usual first order approximations, given a small displacement $y = \eta(t)$ in the position of the surface, the conditions of continuity of pressure and of normal velocity take the form

$$\frac{\partial \phi_1}{\partial t} - \frac{\partial \phi_2}{\partial t} = g\,(\rho_1 - \rho_2)\,\eta \quad ,$$

$$\frac{1}{\rho_1}\frac{\partial \phi_1}{\partial y} - \frac{1}{\rho_2}\frac{\partial \phi_2}{\partial y} = \frac{\partial \eta}{\partial t} \quad ,$$

where the y-axis is the axis normal to the interface.

Although from these equations alone we may only find what sort of surface waves may exist on the interface by prescribing a form for the displacement we shall eliminate the dispersive effects due to gravity by insisting that η be zero for all time. The assumption of an acoustic line source of infinitessimal amplitude and step-function time-dependence in section 1 in no way violates this assertion.

With these remarks in mind we can now state that the results for an (acoustic) source (of delta function time dependence) on the interface arise simply from the formulae in section 1 by differentiating with respect to time throughout. Thus the known results for $\partial\phi/\partial t$ in section 1 represent the values of the velocity potential in the impulse problem.

AF 4561/5

These values are

$$\phi_1 = A\rho_1 c_1^2 \left[t^2 - \left(\frac{r}{c_1}\right)^2\right]^{-1/2} Rl\left\{1 - \frac{i}{mk}\left(\frac{\delta_1^2 - m^2}{1-\delta_1^2}\right)^{1/2}\right\}^{-1}, \qquad \text{for } r < c_1 t,$$

$$\phi_2 = A\rho_1 c_1^2 \left[t^2 - \left(\frac{r}{c_2}\right)^2\right]^{-1/2} Rl\left\{\frac{1}{k} + \left(\frac{1-m^2\delta_2^2}{1-\delta_2^2}\right)^{1/2}\right\}^{-1}, \qquad \text{for } r < c_2 t,$$

where $\delta_1 = \frac{r}{c_1 t}\left\{\cos\theta - i\sin\theta\left[1 - \frac{r^2}{c_1^2 t^2}\right]^{1/2}\right\}^{-1}$, and $\delta_2 = \frac{r}{c_2 t}\left\{\cos\theta - i\sin\theta\left[1 - \frac{r^2}{c_2^2 t^2}\right]^{1/2}\right\}^{-1}$.

Also

$$\phi_2 = A\rho_1 c_1^2 \left[\left(\frac{r}{c_2}\right)^2 - t^2\right]^{-1/2} \left\{\frac{1}{m}\left(\frac{s^{*2} - c_2^2}{c_1^2 - s^{*2}}\right)^{1/2} \Big/ 1 + \frac{1}{m^2k^2}\left(\frac{s^{*2} - c_2^2}{c_1^2 - s^{*2}}\right)\right\},$$

where

$$s^* = \frac{r}{t}\left\{\cos\theta + \sin\theta\left[\frac{r^2}{c_2^2 t} - 1\right]^{1/2}\right\}.$$

This final expression for ϕ_2 is valid within the hyperbolic region GCD.

AF 4561/5

Section 3

The results derived in the acoustic problem are applicable in the theory of electromagnetic pulses involved in the sudden setting-up of a current in an infinite line or of a charged line on the interface between two media. In the former case the vector potential has only one component $A_z = c^2 S(s,\theta)$, the constant k is the ratio of the magnetic permeabilities μ_2/μ_1 and m is the ratio c_2/c_1. The non-zero field components, derived from Maxwell's equations are

$$B_r = \frac{\mu c^2}{r}\frac{\partial S}{\partial \theta}, \quad B_\theta = -\frac{\mu c^2}{t}\frac{\partial S}{\partial s} \quad \text{and} \quad E_z = -s B_\theta$$

For the charged line we relate the quantity S to the scalar potential Φ through the equation $\Phi = c^2 S$. In this case k is the ratio of the dielectric constants ϵ_2/ϵ_1, and the non-zero field components are

$$E_r = -\frac{c^2}{t}\frac{\partial S}{\partial s}, \quad E_\theta = \frac{c^2}{t}\frac{\partial S}{\partial \theta} \quad \text{and} \quad B_z = \frac{1}{t}\frac{\partial S}{\partial \theta}.$$

The results given in equation 16, 17, 19 and 20 may be used directly to derive the field components.

AF 4561/5

CONCLUSION

The assumption of dynamic similarity is used to determine the velocity potential of an impulsive line source which is suddenly set up on the plane which separates two media of different density and sound velocity. The solution for the potential may be identified with Hadamard's elementary solution of the wave equation in the homogeneous case, when

$$
\begin{aligned}
\phi &= \frac{c}{2\pi}\left(c^2t^2 - r^2\right)^{-1/2} && \text{if} \quad r < ct, \\
&= 0 && \text{if} \quad r > ct.
\end{aligned}
$$

In the case of the two media considered in this paper there are similar algebraic singularities on the shock fronts $r = c_1 t$ in medium 1 and $r = c_2 t$ in medium 2.

A feature of the solution is that since $\partial W/\partial f$ is real on the section OC of the interface, there is no normal velocity between the two subsonic regions. This section of the interface is a contact discontinuity (i.e. OC is a steadily expanding vortex-sheet).

The author presented these results verbally at the spring meeting of the U.R.S.I. in Washington, D. C. A similar method has been used independently by Keller and Gardner (1959) to find the solution for a line dipole.

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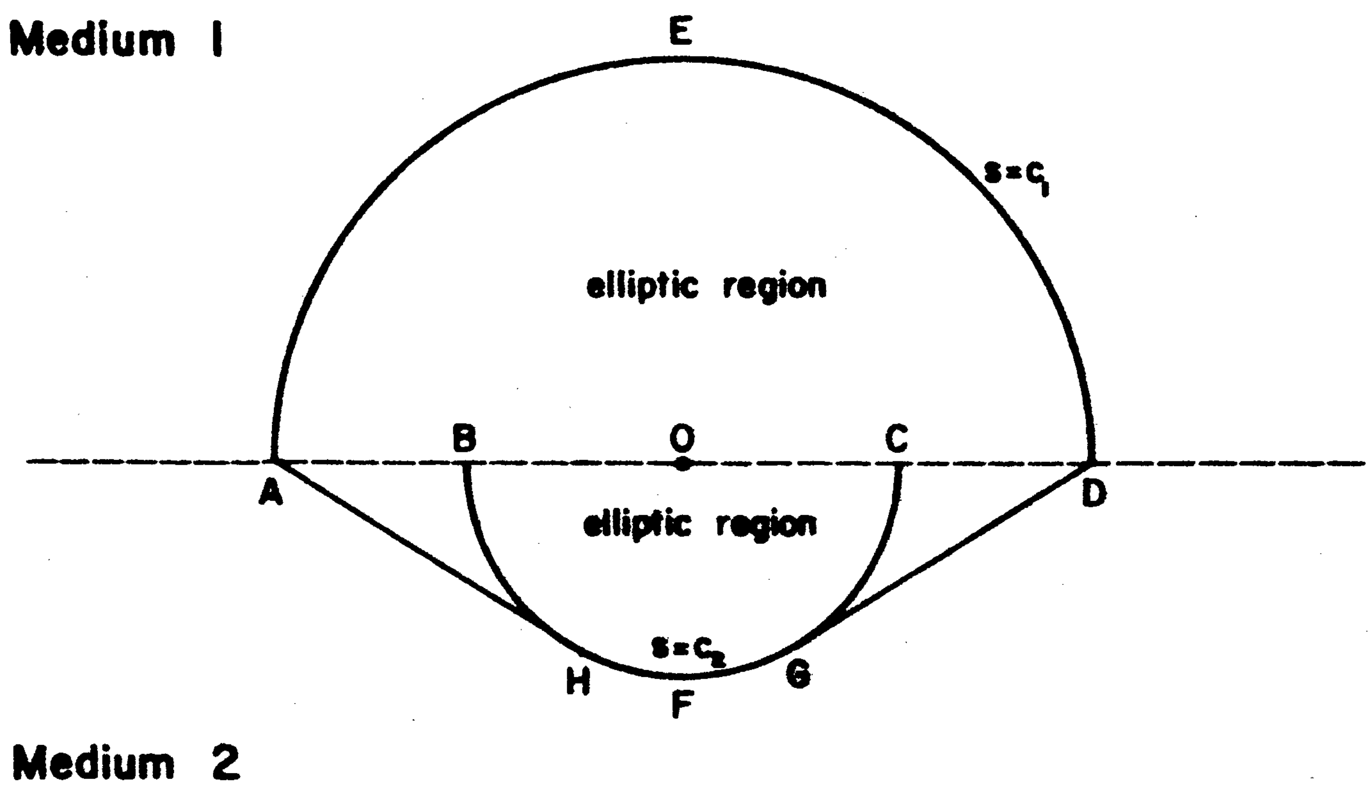

The elliptic regions in the (S, θ) plane.

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